

Deterministic Selection of Phase Sequences in Low Complexity SLM Scheme

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Abstract—Selected mapping (SLM) is a suitable scheme, which can solve the peak-to-average power ratio (PAPR) problem. Recently, many researchers have concentrated on reducing the computational complexity of the SLM schemes. One of the low complexity SLM schemes is the Class III SLM scheme which uses only one inverse fast fourier transform (IFFT) operation for generating one orthogonal frequency division multiplexing (OFDM) signal sequence. By selecting rotations and cyclic shifts randomly, it can generate N^3 alternative OFDM signal sequences, where N is the FFT size. But this selection can not guarantee the optimal PAPR reduction performances. Therefore, in this paper, we propose a simple deterministic cyclic shifts selection method which is optimal in case of having low variance of correlation coefficient between two alternative OFDM signal sequences. And we show that cyclic shifts are highly dependent on the PAPR reduction performance than rotations. For small FFT size and the number of alternative signal sequences is close to $N/8$, simulation results show that the proposed scheme can achieve better PAPR reduction performance than the Class III SLM scheme.

Index Terms—Correlation coefficient, peak-to-average power ratio (PAPR), selected mapping (SLM), variance of correlation.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is one of the most popular multicarrier modulation technique. Because of the orthogonality of subcarriers, the receiver can recover the transmitted data without interferences. Due to this robustness against the multipath channel, OFDM has been adopted as the standards for various wireless communication systems such as IEEE 802.11 a (WLAN), IEEE 802.16 (WiMAX), and long term evolution (LTE). But, it also has the serious drawback which is high peak-to-power ratio (PAPR). When OFDM signals pass through the high power amplifier (HPA), it has in-band distortion and out-of-band radiation. Thus, to reduce the PAPR, several schemes are proposed.

Clipping is the simplest technique, tone reservation reserves the tones for peak canceling signal, and probabilistic method, selected mapping (SLM) and partial transmit sequence (PTS), generate alternative signal sequences and choose the one with the lowest PAPR.

This paper is organized as follows. Section II introduces the Class III SLM scheme. Optimal condition and deterministic

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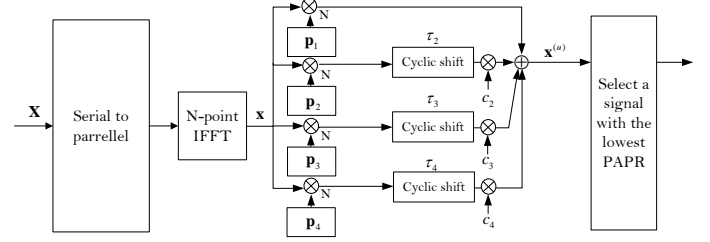


Fig. 1. Block diagram of the Class III SLM scheme

cyclic shifts generation method are proposed in Section III. In Section IV, simulation results are shown and Section V concludes this paper.

II. CLASS III SLM SCHEME

The Class III SLM scheme is introduced by Li *et al.* [1] and it only uses one inverse fast fourier transform (IFFT) operation for generating one orthogonal frequency division multiplexing (OFDM) signal sequence. Fig. 1 shows the block diagram of Class III SLM scheme. Quadrature phase-shift keying (QPSK) or M -ary quadrature amplitude modulation (M -QAM) modulated input symbol sequence, $\mathbf{X} = [X_0, X_1, X_2, \dots, X_{N-1}]$, is the input of IFFT operation, where N is the FFT size. Then, OFDM symbol sequence, $\mathbf{x} = [x_0, x_1, x_2, \dots, x_{N-1}]$, performs N -point circular convolution with 4 base vectors. The 4 base vectors are defined as

$$\begin{aligned} \mathbf{p}_1 &= \left[\underbrace{1, 0, \dots, 0}_{N/4}, \underbrace{1, 0, \dots, 0}_{N/4}, \underbrace{1, 0, \dots, 0}_{N/4}, \underbrace{1, 0, \dots, 0}_{N/4} \right] \\ \mathbf{p}_2 &= \left[\underbrace{1, 0, \dots, 0}_{N/4}, \underbrace{j, 0, \dots, 0}_{N/4}, \underbrace{-1, 0, \dots, 0}_{N/4}, \underbrace{-j, 0, \dots, 0}_{N/4} \right] \\ \mathbf{p}_3 &= \left[\underbrace{1, 0, \dots, 0}_{N/4}, \underbrace{-1, 0, \dots, 0}_{N/4}, \underbrace{1, 0, \dots, 0}_{N/4}, \underbrace{-1, 0, \dots, 0}_{N/4} \right] \\ \mathbf{p}_4 &= \left[\underbrace{1, 0, \dots, 0}_{N/4}, \underbrace{-j, 0, \dots, 0}_{N/4}, \underbrace{-1, 0, \dots, 0}_{N/4}, \underbrace{j, 0, \dots, 0}_{N/4} \right]. \end{aligned} \quad (1)$$

Then, each sequence is right cyclic shifted, τ_i , and rotated, c_i , to generate alternative signal sequences, $\mathbf{x}^{(u)}$. Finally, select the one with the lowest PAPR. The conversion vector for

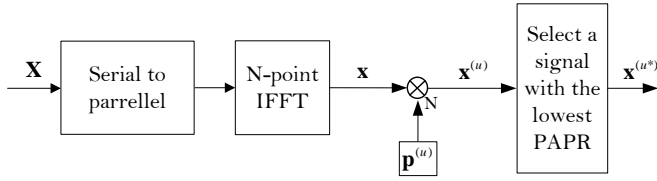


Fig. 2. Block diagram of equivalent Class III SLM scheme

generating u th alternative signal sequence has the general form

$$\mathbf{p}^{(u)} = \sum_{i=1}^4 c_i^{(u)} \mathbf{p}_{i(\tau_i^{(u)})} \quad (2)$$

where $0 \leq \tau_i \leq N/4 - 1$, $c_i \in \{\pm 1, \pm j\}$, $c_i^{(u)}$ is i th rotation value, and $\mathbf{p}_{i(\tau_i^{(u)})}$ is $\tau_i^{(u)}$ right cyclic shifted version of i th base vector, $i = 1, 2, 3$, and 4. By (2), the Class III SLM can generate N^3 alternative signal sequences. Fig. 2 shows the equivalent Class III SLM scheme which is expressed using (2) and it is identical to Fig. 1.

III. DETERMINISTIC SELECTION OF CYCLIC SHIFTS

A. Correlation Analysis

Heo *et al.* [2] showed that the correlation coefficient, ρ_{ij} , between sample powers of two alternative OFDM signal sequences is approximated as follows

$$\rho_{ij}(\tau) \simeq \frac{1}{N^2} \left| \sum_{k=0}^{N-1} P^{(i)}(k) P^{(j)*}(k) e^{-j \frac{2\pi k}{N} \tau} \right|^2 \quad (3)$$

where $P^{(i)}(k)$ is k th element in i th phase sequence, $(\cdot)^*$ denotes complex conjugate operation, and $0 \leq \tau \leq N - 1$. Let $\mathbf{P}_1 = \text{FFT}\{\mathbf{p}_1\}$. Then, FFT outputs of 4 base vectors in (1) are expressed as

$$\begin{aligned} \mathbf{P}_1 &= 4 \begin{bmatrix} \underbrace{1, 0, 0, 0}_4, \underbrace{1, 0, 0, 0}_4, \dots, \underbrace{1, 0, 0, 0}_4 \end{bmatrix} \\ \mathbf{P}_2 &= 4 \begin{bmatrix} \underbrace{0, 1, 0, 0}_4, \underbrace{0, 1, 0, 0}_4, \dots, \underbrace{0, 1, 0, 0}_4 \end{bmatrix} \\ \mathbf{P}_3 &= 4 \begin{bmatrix} \underbrace{0, 0, 1, 0}_4, \underbrace{0, 0, 1, 0}_4, \dots, \underbrace{0, 0, 1, 0}_4 \end{bmatrix} \\ \mathbf{P}_4 &= 4 \begin{bmatrix} \underbrace{0, 0, 0, 1}_4, \underbrace{0, 0, 0, 1}_4, \dots, \underbrace{0, 0, 0, 1}_4 \end{bmatrix}. \end{aligned} \quad (4)$$

Because circular convolution in time domain is identical to component wise multiplication in frequency domain, $\mathbf{x} \oplus_N \mathbf{p}_1 = \mathbf{X} \odot \mathbf{P}_1$, input symbol sequence is multiplied by \mathbf{P}_i , $i = 1, 2, 3$, and 4. It means that the 4 vectors, \mathbf{P}_i , are partitioning vectors same as the partitioning in the PTS. For example, \mathbf{P}_1 has all zero elements except when its indices are $0 \bmod 4$. Thus, $\mathbf{X} \odot \mathbf{P}_1 = [X_0, 0, 0, 0, X_4, 0, 0, 0, \dots, X_{N-4}, 0, 0, 0]$. It can also be applied to other partitioning vectors. After all, the Class III SLM scheme can be interpreted as an interleaved partitioned PTS scheme [5].

u th alternative signal sequence in the Class III SLM scheme can be expressed as

$$\begin{aligned} \mathbf{x}^{(u)} &= \mathbf{p}^{(u)} \otimes_N \mathbf{x} \\ &= \left(\sum_{i=1}^4 c_i^{(u)} \mathbf{p}_{i(\tau_i^{(u)})} \right) \otimes_N \mathbf{x} \\ &= \text{IFFT} \left\{ \left(\sum_{i=1}^4 c_i^{(u)} \mathbf{P}_i(\tau_i^{(u)}) \right) \odot \mathbf{X} \right\} \end{aligned} \quad (5)$$

where $\mathbf{P}_i(\tau_i^{(u)})$ denotes the linear phase in frequency domain by cyclic shift, $\tau_i^{(u)}$, in time domain. And then, we can consider $\sum_{i=1}^4 c_i^{(u)} \mathbf{P}_i(\tau_i^{(u)})$ in (5) as a phase sequence in SLM scheme. Thus, u th phase sequence in the Class III SLM scheme can be expressed as follows

$$\mathbf{P}^{(u)} = \sum_{i=1}^4 c_i^{(u)} \mathbf{P}_i(\tau_i^{(u)}). \quad (6)$$

For $i = 1$ in (6), it can be computed as

$$c_1^{(u)} \mathbf{P}_1(\tau_1^{(u)}) = \left[\underbrace{c_1^{(u)}, 0, 0, 0}_4, \dots, \underbrace{c_1^{(u)} e^{-j \frac{2\pi(N-4)\tau_1^{(u)}}{N}}, 0, 0, 0}_4 \right]. \quad (7)$$

With this result, we can compute the u th phase sequence as (8), where the elements are

$$P^{(u)}(k) = \begin{cases} c_1^{(u)} e^{-j \frac{2\pi k \tau_1^{(u)}}{N}}, & k = 0 \bmod 4 \\ c_2^{(u)} e^{-j \frac{2\pi k \tau_2^{(u)}}{N}}, & k = 1 \bmod 4 \\ c_3^{(u)} e^{-j \frac{2\pi k \tau_3^{(u)}}{N}}, & k = 2 \bmod 4 \\ c_4^{(u)} e^{-j \frac{2\pi k \tau_4^{(u)}}{N}}, & k = 3 \bmod 4. \end{cases} \quad (9)$$

Finally, we can substitute (8) to (3) and then consider only inner parts of the magnitude $|\cdot|$. We can replace it as $A(\tau)$ and then we can get 4 terms. We can also replace each term with $A_{(i)}(\tau)$, $i = 1, 2, 3$, and 4, as follows

$$\begin{aligned} A(\tau) &= \sum_{k=0}^{N-1} \mathbf{P}^{(i)}(k) \mathbf{P}^{(j)*}(k) e^{-j \frac{2\pi k}{N} \tau} \\ &= \sum_{v=0}^{\frac{N}{4}-1} c_1^{(i)} c_1^{(j)*} e^{-j \frac{2\pi(4v)}{N} (\tau + \tau_1^{(i)} - \tau_1^{(j)})} \\ &\quad + \sum_{v=0}^{\frac{N}{4}-1} c_2^{(i)} c_2^{(j)*} e^{-j \frac{2\pi(4v+1)}{N} (\tau + \tau_2^{(i)} - \tau_2^{(j)})} \\ &\quad + \sum_{v=0}^{\frac{N}{4}-1} c_3^{(i)} c_3^{(j)*} e^{-j \frac{2\pi(4v+2)}{N} (\tau + \tau_3^{(i)} - \tau_3^{(j)})} \\ &\quad + \sum_{v=0}^{\frac{N}{4}-1} c_4^{(i)} c_4^{(j)*} e^{-j \frac{2\pi(4v+3)}{N} (\tau + \tau_4^{(i)} - \tau_4^{(j)})} \\ &= A_{(1)}(\tau) + A_{(2)}(\tau) + A_{(3)}(\tau) + A_{(4)}(\tau). \end{aligned} \quad (10)$$

$$\mathbf{P}^{(u)} = \sum_{i=1}^4 c_i^{(u)} \mathbf{P}_i(\tau_i^{(u)})$$

$$= \left[c_1^{(u)}, c_2^{(u)} e^{-j \frac{2\pi \tau_2^{(u)}}{N}}, c_3^{(u)} e^{-j \frac{2\pi \tau_3^{(u)}}{N}}, c_4^{(u)} e^{-j \frac{2\pi \tau_4^{(u)}}{N}}, \dots, c_1^{(u)} e^{-j \frac{2\pi (N-4)\tau_2^{(u)}}{N}}, c_2^{(u)} e^{-j \frac{2\pi (N-3)\tau_3^{(u)}}{N}}, c_3^{(u)} e^{-j \frac{2\pi (N-2)\tau_4^{(u)}}{N}}, c_4^{(u)} e^{-j \frac{2\pi (N-1)\tau_1^{(u)}}{N}} \right] \quad (8)$$

TABLE I
 $\bar{A}_{(i)}(\tau)$ VALUES FOR DIFFERENT τ .

τ	$-d_{\tau_i} \bmod N$	$\frac{N}{4} - d_{\tau_i}$	$\frac{2N}{4} - d_{\tau_i}$	$\frac{3N}{4} - d_{\tau_i}$
$\bar{A}_{(1)}(\tau)$	$\frac{N}{4}$	$\frac{N}{4}$	$\frac{N}{4}$	$\frac{N}{4}$
$\bar{A}_{(2)}(\tau)$	$\frac{N}{4}$	$\frac{N}{4}j$	$\frac{N}{4}$	$-\frac{N}{4}j$
$\bar{A}_{(3)}(\tau)$	$\frac{N}{4}$	$-\frac{N}{4}$	$\frac{N}{4}$	$-\frac{N}{4}$
$\bar{A}_{(4)}(\tau)$	$\frac{N}{4}$	$-\frac{N}{4}j$	$\frac{N}{4}$	$\frac{N}{4}j$

Let us consider one $A_{(i)}(\tau)$. And take the rotation $c_1^{(i)} c_1^{(j)*}$ out of summation, then we can get $\bar{A}_{(1)}(\tau)$.

$$\begin{aligned} A_{(1)}(\tau) &= \sum_{v=0}^{\frac{N}{4}-1} c_1^{(i)} c_1^{(j)*} e^{-j \frac{2\pi(4v)}{N} (\tau + \tau_1^{(i)} - \tau_1^{(j)})} \\ &= c_1^{(i)} c_1^{(j)*} \sum_{v=0}^{\frac{N}{4}-1} e^{-j \frac{2\pi(4v)}{N} (\tau + \tau_1^{(i)} - \tau_1^{(j)})} \\ &= c_1^{(i)} c_1^{(j)*} \sum_{v=0}^{\frac{N}{4}-1} e^{-j \frac{2\pi(v)}{N/4} (\tau + \tau_1^{(i)} - \tau_1^{(j)})} \\ &= c_1^{(i)} c_1^{(j)*} \bar{A}_{(1)}(\tau). \end{aligned} \quad (11)$$

In the same manner, the rest can also be computed as

$$\begin{aligned} \bar{A}_{(2)}(\tau) &= \sum_{v=0}^{\frac{N}{4}-1} e^{-j \frac{2\pi(v+1/4)}{N/4} (\tau + \tau_2^{(i)} - \tau_2^{(j)})} \\ \bar{A}_{(3)}(\tau) &= \sum_{v=0}^{\frac{N}{4}-1} e^{-j \frac{2\pi(v+2/4)}{N/4} (\tau + \tau_3^{(i)} - \tau_3^{(j)})} \\ \bar{A}_{(4)}(\tau) &= \sum_{v=0}^{\frac{N}{4}-1} e^{-j \frac{2\pi(v+3/4)}{N/4} (\tau + \tau_4^{(i)} - \tau_4^{(j)})}. \end{aligned} \quad (12)$$

If the value of $(\tau + \tau_1^{(i)} - \tau_1^{(j)})$ is multiple of $N/4$, $\bar{A}_{(1)}(\tau)$ becomes $N/4$. Otherwise, it becomes zero since the sum of all points in an unit circle is zero. Let $d_{\tau_v} = \tau_v^{(i)} - \tau_v^{(j)}$, $v = 1, 2, 3$, and 4, be the difference between two cyclic shifts. Then, we can calculate all possible values of $\bar{A}_{(i)}(\tau)$ with different τ . The results are shown in Table I. $\bar{A}_{(i)}(\tau)$ can have 4 possible values each. Therefore, if four $\bar{A}_{(i)}(\tau)$ have values without sharing τ , there can be 16 values for different τ .

B. Optimal condition for low variance of correlation

In [3], authors showed that phase sequence sets with low variance of correlation have better PAPR reduction performance. Based on this results, we found the condition which

TABLE II
 DETERMINISTIC GENERATION OF PHASE SEQUENCES.

u_i	1	2	3	\dots	k
τ_2	1	2	3	\dots	$k \bmod \frac{N}{4}$
τ_3	2	4	6	\dots	$2k \bmod \frac{N}{4}$
τ_4	3	6	9	\dots	$3k \bmod \frac{N}{4}$

can make the variance of correlation coefficient low.

$$\text{Optimal condition : } d_{\tau_1} \neq d_{\tau_2} \neq d_{\tau_3} \neq d_{\tau_4} \bmod \frac{N}{4}. \quad (13)$$

That is, to achieve optimal PAPR reduction performance, we have to choose the cyclic shifts with different d_{τ_v} between any $\binom{U}{2}$ alternative signal sequence pairs, where U is the total number of alternative signal sequences. Suppose 2 or more $\bar{A}_{(i)}(\tau)$ have the values in Table I at same τ' . By Parseval's Theorem, the magnitude should be larger than a case which have only one signal at τ' . Therefore, sharing same τ' might increase the variance of correlation and it is not desirable [3].

C. Deterministic Cyclis Shifts Generation Method

In this subsection, we propose a simple cyclic shifts generation method. Table II shows the cyclic shifts for each τ_i , where u denotes the alternative signal sequence index. τ_2 has the values which are multiple of 1 at an increasing order. Similarly, τ_3 has multiple of 2 and τ_4 has multiple of 3. For any u_1, u_2 , where $u_1 \neq u_2$ and $u_1 < u_2$, the corresponding differences are $d_{\tau_1} = 0$, $d_{\tau_2} = u_2 - u_1$, $d_{\tau_3} = 2(u_2 - u_1)$, and $d_{\tau_4} = 3(u_2 - u_1) = 2(u_2 - u_1) + (u_2 - u_1)$, respectively. If $u_2 - u_1 = \frac{N}{8}$, $d_{\tau_2} = d_{\tau_4} \bmod \frac{N}{4}$. Therefore, $u_2 - u_1 < \frac{N}{8}$. Now, we have show that $d_{\tau_i} - d_{\tau_j} = k(u_2 - u_1) \neq 0 \bmod \frac{N}{4}$, where $i, j \in \{1, 2, 3, 4\}$, $i < j$, and $k = 1, 2$, and 3. When $k = 1$ and 2, this condition holds by $u_1 < u_2$ and $u_2 - u_1 < \frac{N}{8}$. And when $k = 3$, because of $N = 2^m$, where m is natural number, $3(u_2 - u_1) \neq 0 \bmod \frac{N}{4} = 2^{m-2}$. Thus, when $u_2 - u_1 = \frac{N}{8}$, it always satisfy the optimal condition and the maximum number of alternative signal sequences is $\frac{N}{8}$.

IV. SIMULATION RESULT

In this section, we compare the PAPR reduction performance between Class III SLM (C3-SLM) scheme and the proposed deterministic selection SLM scheme (DS-SLM). PAPR reduction performance is evaluated by complementary cumulative distribution function (CCDF) which shows a probability that is larger than a certain threshold level, γ . In C3-SLM scheme, cyclic shift and rotation values are randomly

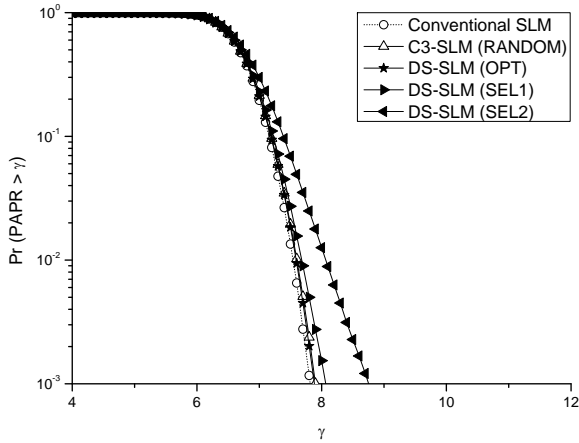


Fig. 3. PAPR reduction performance when 16-QAM, $N = 256$, and $U = 10$.

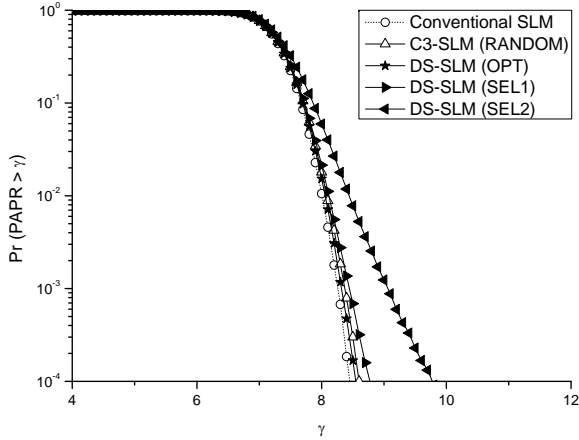


Fig. 4. PAPR reduction performance when 16-QAM, $N = 512$, and $U = 10$.

generated. While in DS-SLM scheme, cyclic shift values are taken from Table II and rotation values are all 1, that is, $c_2 = c_3 = c_4 = 1$. Also, $d_{\tau_1} = d_{\tau_2}$ and $d_{\tau_1} = d_{\tau_2} = d_{\tau_3}$ cases are considered for comparison. We call the C3-SLM with random generation of rotation and cyclic shift values as C3-SLM (RANDOM), DS-SLM with optimal condition as DS-SLM (OPT), DS-SLM with $d_{\tau_1} = d_{\tau_2}$ case as DS-SLM (SEL1), and DS-SLM with $d_{\tau_1} = d_{\tau_2} = d_{\tau_3}$ case as DS-SLM (SEL2), respectively. Figs. (3)–(5) show the PAPR reduction performance of C3-SLM and DS-SLM scheme, when 16-QAM, $U = 10$, and with different FFT size. Conventional SLM has the best PAPR reduction performance in all figures. Because of the correlation in C3-SLM scheme, it can not achieve better PAPR reduction performance than conventional SLM. When $N = 256$, C3-SLM (RANDOM) and DS-SLM (OPT) can achieve almost same PAPR reduction performance. Since DS-SLM (SEL1) and DS-SLM (SEL2) can not satisfy the optimality condition, PAPR reduction performances are degraded than DS-SLM (OPT). This result shows that d_{τ_i} is highly dependent on the PAPR reduction performance than rotation values. When $N = 512$, DS-SLM (OPT) can achieve better PAPR reduction performance than C3-SLM (RANDOM) slightly, and when $N = 1024$, C3-SLM (RANDOM) has almost same PAPR reduction performance as DS-SLM (SEL1) which is degraded than DS-SLM (OPT). Therefore,

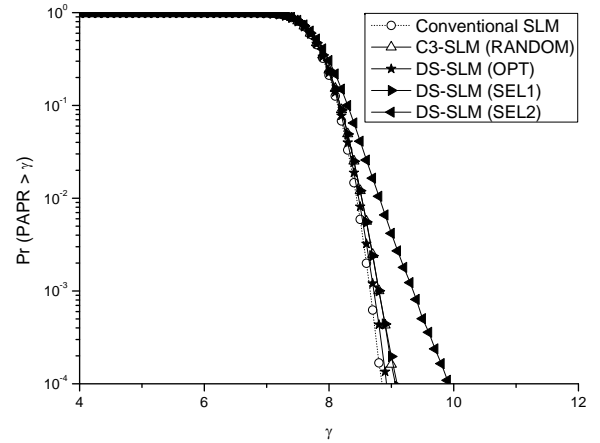


Fig. 5. PAPR reduction performance when 16-QAM, $N = 1024$, and $U = 10$.

when N is large, DS-SLM (OPT) can achieve better PAPR reduction performance than C3-SLM (RANDOM).

V. CONCLUSION

In this paper, we propose a simple deterministic cyclic shifts generation method which satisfy the optimal condition and show that cyclic shifts are highly dependent on the PAPR reduction performance than rotations. Simulation results show that the proposed DS-SLM scheme can achieve better PAPR reduction performance than the C3-SLM scheme for small FFT size and U is close to $N/8$.

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